

Influence of several irreversible losses on the performance of a ferroelectric Stirling refrigeration-cycle

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Abstract

An irreversible cycle model of the Stirling refrigeration-cycle, using a ferroelectric material as the working substance, is established. Several irreversibilities due to thermal resistances between the working substance and the heat reservoirs, regenerative losses in two regenerative processes, and heat-leak loss between the heat reservoirs are taken into account. The influence of these irreversible losses on the performance of the ferroelectric Stirling refrigeration-cycle is analyzed, based on a general expression of the polarization of ferroelectric materials and a linear heat-transfer law. The cooling rate is optimized for a given power input. Some fundamental optimal relations and general performance characteristic curves of the cycle are obtained. The maximum cooling rate and other relevant performance parameters are determined. Some special cases are discussed in detail. When the regenerative losses are neglected, the results obtained may be directly used to describe the optimal performance of a ferroelectric Carnot refrigeration-cycle. Moreover, it is expounded that the calculated results are very general and also suitable for the ferroelectric Stirling and Carnot refrigeration cycles whose working substances obey the Curie–Weiss law and Curie law. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

Since the early 1970s, many researchers have been searching for more environment-friendly refrigerants and new advanced cycle modes [1–4]. Ferroelectric refrigerators use no environmentally sensitive materials and consequently have received increasing attention [5–10]. The previous investigations were mainly concentrated on the

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Nomenclature

a	$= Q_L / \Delta s$, defined in Eq. (20)
b	$= (1 - \eta) C_{PA} / \Delta s$, defined in Eq. (20)
b_1	$= 2u / \Delta s$, defined in Eq. (21)
C	Curie constant
C_P	specific heat at constant electrical polarization
C_{PA}	average value of C_P
d	$= \sqrt{(1+b)(b+e)T_c/T_h - be}$, defined in Eq. (29)
E	electric-field strength
E_i	electric-field strength in the state i ($= a, b, c, d$)
e	$= b_1 k T_c$, defined in Eq. (27)
$F(P)$	function determined by Eq. (1)
h	$= \sqrt{k_1/k_2}$
k	$= 1/(1/\sqrt{k_1} + 1/\sqrt{k_2})^2$, defined in Eq. (27)
k_B	Boltzmann constant
k_L	coefficient of heat leak
k_1	thermal conductance between the working substance and the heat sink
k_2	thermal conductance between the working substance and the cooled space
N	number of electric dipoles per unit volume
P	electrical polarization
P_i	electric polarization in the state i ($= a, b, c, d$)
p	power input
p_r	power input at maximum cooling rate
p_ε	power input at zero coefficient-of-performance
p^*	dimensionless power input
Q_c	net heat absorbed from the cooled space per cycle
Q_h	net heat released to the heat sink per cycle
Q_L	heat leak loss per cycle
Q_r	regenerative losses per cycle
Q_1	heat released to the heat sink in the high-temperature isothermal process
Q_2	heat absorbed from the cooled space in the low-temperature isothermal process
Q_{12}	heat transferred into the regenerator per cycle
Q_{34}	heat transferred out of the regenerator per cycle
q_L	rate of the heat leak
r	cooling rate
$r_{\max}$	maximum cooling rate
r^*	dimensionless cooling rate
s	entropy of ferroelectric materials per unit volume
s_0	entropy of ferroelectric materials per unit volume when $P=0$

s_i	entropy of ferroelectric materials per unit volume in the state i ($= a, b, c, d$)
Δs_1	entropy change of ferroelectric materials per unit volume in the high-temperature isothermal process
Δs_2	entropy change of ferroelectric materials per unit volume in the low-temperature isothermal process
T	absolute temperature
T_c	temperature of the cooled space
T_h	temperature of the heat sink
T_0	Curie temperature
T_1	temperature of the working substance in the high-temperature isothermal process
T_2	temperature of the working substance in the low-temperature isothermal process
T_{1r}	value of T_1 at maximum cooling rate
T_{2r}	value of T_2 at maximum cooling rate
t	cycle period
t_1, t_2, t_3, t_4	times spent on two isothermal and two regenerative processes respectively
u	a proportional constant appearing in Eq. (18)
v	Lagrange multiplier
y	$= T_1/T_2$
β	a parameter which is independent of temperature
ε	coefficient of performance
ε_r	coefficient of performance at maximum cooling rate
η	efficiency of the regenerator
μ	moment of electric dipole

ferroelectric Carnot cycle. Like the magnetic Stirling refrigeration cycle, it is expected that the ferroelectric Stirling refrigeration-cycle will play an important role in refrigeration in the near future.

The Stirling refrigeration-cycle is one of the important cycle models of refrigerators with regeneration. According to the theory of classical thermodynamics, the Stirling refrigeration-cycle may possess the condition of perfect regeneration through the use of a reversible regenerator. Its coefficient-of-performance is equal to that of a reversible Carnot refrigeration-cycle. However, when the irreversibility of the heat transfer is considered, the performance of the Stirling refrigeration cycle is different from that of a Carnot refrigeration-cycle [11–13]. In recent years, although the influence of irreversibilities on the performance of the Stirling refrigeration cycle using a gas or magnetic material as the working substance has been investigated extensively, the influence of irreversibilities on the performance of the ferroelectric Stirling refrigeration-cycle has been rarely studied.

In the present paper, an irreversible cycle model of the ferroelectric Stirling refrigerator will be established and used to investigate the influence of irreversibilities due to finite-rate heat transfers between the working substance and the heat reservoirs, regenerative losses in two regenerative processes, and heat-leak loss between the heat reservoirs on the performance of the cycle. The results obtained here will be of real significance.

2. Thermodynamic properties of ferroelectric materials

According to the theory of the Boltzmann statistics, the ferroelectric polarization may be given by [14,15]

$$P = N\mu \tanh \frac{\mu(E + \beta P)}{k_B T}, \quad (1)$$

where N is the number of the electric dipoles per unit volume, μ is the moment of an electric dipole, k_B is the Boltzmann constant, and β is a constant which is independent of temperature, E is the electric-field strength, and T is the absolute temperature.

Using Eq. (1), one can prove [10] that the entropy of ferroelectric materials per unit volume and the specific heat at constant electrical polarization are, respectively, given by

$$s = s_0(T) - P(E + \beta P)/T + Nk \ln \cosh \frac{\mu(E + \beta P)}{k_B T} \quad (2)$$

and

$$C_p = C_P(T), \quad (3)$$

where $s_0(T)$ is the entropy of ferroelectric materials per unit volume when $P = 0$ and is a function only of temperature. Its value depends on the properties of the specific ferroelectric material. Eq. (3) shows clearly that the specific heat at constant electrical polarization is a function only of temperature while independent of the properties of the ferroelectric materials.

3. An irreversible ferroelectric Stirling refrigeration-cycle

When a ferroelectric material is used as the working substance, the Stirling refrigeration-cycle consists of two isothermal and two constant electrical polarization processes, as shown in Fig. 1, where T_h and T_c are the temperatures of the heat sink and the cooled space, T_1 and T_2 are the temperatures of the working substance in the high- and low-temperature isothermal processes, Q_1 and Q_2 are the amounts of heat exchanged between the working substance and the heat reservoirs during two isothermal processes, Q_{12} and Q_{34} are the amounts of heat exchanged between the

working substance and the regenerator during two constant electrical polarization processes.

When the cycle is reversible, the temperatures T_1 and T_2 of the working substance in two isothermal processes are identical with the temperatures T_h and T_c of the heat reservoirs, respectively. Using Eq. (2), one can calculate that the amounts of heat released from and absorbed by the working substance during the high- and low-temperature isothermal processes are, respectively, given by

$$Q_1 = T_1(s_a - s_b) = T_1 \Delta s_1 \quad (4)$$

and

$$Q_2 = T_2(s_d - s_c) = T_2 \Delta s_2 \quad (5)$$

where

$$\Delta s_1 = \frac{P_b(E_b + \beta P_b) - P_a(E_a + \beta P_a)}{T_1} + Nk_B \ln \frac{\cosh[\mu(E_a + \beta P_a)/(k_B T_1)]}{\cosh[\mu(E_b + \beta P_b)/(k_B T_1)]} \quad (6)$$

$$\Delta s_2 = \frac{P_c(E_c + \beta P_c) - P_d(E_d + \beta P_d)}{T_2} + Nk_B \ln \frac{\cosh[\mu(E_d + \beta P_d)/(k_B T_2)]}{\cosh[\mu(E_c + \beta P_c)/(k_B T_2)]} \quad (7)$$

The parameters P_i and E_i are the ferroelectric polarizations and the electric field strengths in the states i ($= a, b, c, d$) shown in Fig. 1.

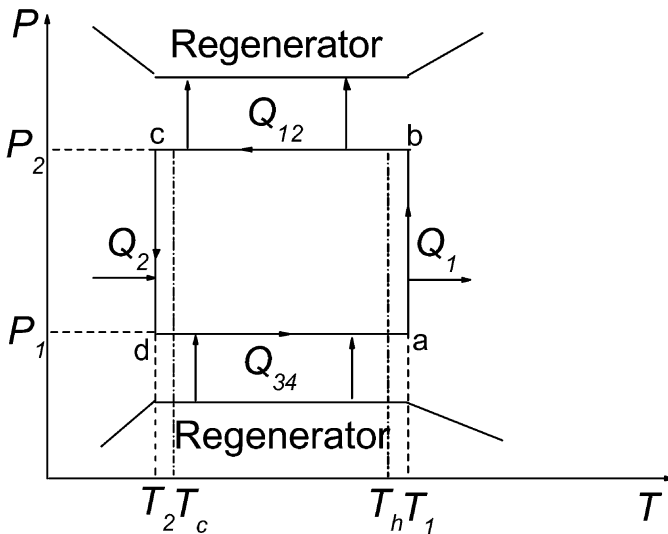


Fig. 1. The P-T diagram of a ferroelectric Stirling refrigeration-cycle.

From Eq. (1), one can derive

$$(E + \beta P)/T = F(P), \quad (8)$$

where $F(P)$ is a function only of the polarization and determined by Eq. (1). For the process of a given polarization P , $F(P)$ is a constant, so that

$$(E + \beta P)/T = \text{const.} \quad (9)$$

According to Eq. (9) and Fig. 1, one can find without difficulty that $\Delta s_1 = \Delta s_2 = \Delta s$ is a constant, which is independent of the electric field strength E and the temperature T . Thus, only if the change of the electric field strength with the temperature is controlled properly, can the constant electric polarization process be realized.

For two constant electric polarization processes, it has been shown [10] that

$$Q_{12} = Q_{34}, \quad (10)$$

which may be derived from Eq. (3). This shows clearly that a ferroelectric Stirling refrigeration-cycle may possess the condition of perfect regeneration and can attain perfect regeneration through the use of a perfect regenerator.

When the irreversibility of heat transfer is taken into account, the cycle is irreversible. The temperatures T_1 and T_2 of the working substance in the high- and low-temperature isothermal processes are, respectively, different from the temperatures T_h and T_c of the heat sink and the cooled space, but Eqs. (4) and (5) are still true. When the heat transfer obeys a linear heat-transfer law [16–20], Q_1 and Q_2 may be expressed as

$$Q_1 = k_1(T_1 - T_h)t_1 \quad (11)$$

and

$$Q_2 = k_2(T_c - T_2)t_2, \quad (12)$$

where k_1 and k_2 are the thermal conductances between the working substance and the heat reservoirs at temperatures T_h and T_c , and t_1 and t_2 are the times taken for the two isothermal processes at temperatures T_1 and T_2 .

For an irreversible ferroelectric Stirling refrigeration cycle, not only the isothermal processes but also the regenerative processes are affected by thermal resistances, so that the cycle cannot attain the perfect regeneration. Usually, it may be assumed that the regenerative losses Q_r per cycle are given by

$$Q_r = (1 - \eta) \int_{T_2}^{T_1} C_P dT, \quad (13)$$

where η is the efficiency of the regenerator. In general, C_P is closely dependent on the temperature. In order to simplify the calculation and analyze the main characteristics

of regenerative influence, C_P may be substituted by the average value C_{PA} of the specific heats at constant electric polarization between T_1 and T_2 , so that Eq. (13) may be simply written as

$$Q_r = (1 - \eta)C_{PA}(T_1 - T_2). \quad (14)$$

Because a ferroelectric Stirling refrigeration cycle is only operated between two heat reservoirs at temperatures T_h and T_c , the regenerative losses Q_r must be compensated from the heat sink during one constant electrical polarization process and released to the cooled space during the other constant electrical polarization process. If not, the regenerator would not work normally.

Because the ferroelectric Stirling refrigeration-cycle is operated between two heat reservoirs, there naturally exists a heat leak from the heat sink at temperature T_h to the cooled space at temperature T_c . It is often assumed [21–23] that the heat-leak loss is directly proportional to the temperature difference between the two heat reservoirs and the cycle time, i.e.

$$Q_L = k_L(T_h - T_c)t \quad (15)$$

where k_L is the coefficient of the heat leak and t is the cycle period.

Thus, the net amounts of heat released to the heat sink and absorbed from the cold reservoir per cycle are

$$Q_h = Q_1 - Q_r - Q_L \quad (16)$$

and

$$Q_c = Q_2 - Q_r - Q_L. \quad (17)$$

Owing to the influence of thermal resistances, the regenerative processes need a non-negligible time compared with the time of the isothermal processes. It is often assumed that the time of the regenerative processes is directly proportional to the temperature difference of the working substance in the two isothermal processes, i.e.,

$$t_3 = t_4 = u(T_1 - T_2), \quad (18)$$

where u is a proportionality constant which is independent of temperature. Therefore, the cycle period is given by

$$t = t_1 + t_2 + 2t_3 \quad (19)$$

Such a cycle model includes the influence of thermal resistances between the working substance and the heat reservoirs, the regenerative losses in the regenerative processes and the heat leak loss between the heat reservoirs. It may be used to discuss the influence of several key irreversibilities on the performance of a ferroelectric Stirling cycle and obtain some optimal results.

4. General performance characteristics

Using the above equations, one can obtain the expressions of the important performance parameters of the ferroelectric Stirling refrigeration cycle. For example, the coefficient of performance, cooling rate, and power input are, respectively, given by

$$\varepsilon = \frac{Q_c}{Q_h - Q_c} = \frac{Q_2 - Q_r - Q_L}{Q_1 - Q_2} = \frac{T_2 - a}{T_1 - T_2} - b, \quad (20)$$

$$r = \frac{Q_c}{t} = \frac{T_2 - b(T_1 - T_2)}{\frac{T_1}{k_1(T_1 - T_h)} + \frac{T_2}{k_2(T_c - T_2)} + b_1(T_1 - T_2)} - q_L, \quad (21)$$

and

$$p = \frac{Q_h - Q_c}{t} = \frac{T_1 - T_2}{\frac{T_1}{k_1(T_1 - T_h)} + \frac{T_2}{k_2(T_c - T_2)} + b_1(T_1 - T_2)} \quad (22)$$

where $a = Q_L/\Delta s$, $b = (1 - \eta)C_{PA}/\Delta s$, $b_1 = 2u/\Delta s$, and $q_L = k_L(T_h - T_c)$ is the rate of the heat leak.

Using a refrigerator, one always wants to obtain a cooling rate as large as possible for a given power input. For this purpose, we introduce the Lagrangian

$$L = r + vp = \frac{1 + (v - b)(y - 1)}{\frac{y}{k_1(yT_2 - T_h)} + \frac{1}{k_2(T_c - T_2)} + b_1(y - 1)} - q_L, \quad (23)$$

where v is a Lagrange multiplier and $y = T_1/T_2$.

Using the extremal condition

$$\frac{\partial L}{\partial T_2} = 0 \quad (24)$$

and Eq. (23), one can obtain an optimal relation as

$$y(T_c - T_2) = h(yT_2 - T_h), \quad (25)$$

where $h = \sqrt{k_1/k_2}$. Substituting Eq. (25) into Eqs. (20)–(22) and eliminating T_2 , one has

$$\varepsilon = \frac{T_c y + h(T_h - ay) - ay}{(y - 1)(T_c y + hT_h)} - b \quad (26)$$

$$r = kT_c \frac{1 - b(y - 1)}{y/(y - T_h/T_c) + e(y - 1)} - q_L, \quad (27)$$

and

$$p = kT_c \frac{y - 1}{y/(y - T_h/T_c) + e(y - 1)}, \quad (28)$$

where $k = 1/(1/\sqrt{k_1} + 1/\sqrt{k_2})^2$ and $e = b_1 k T_c$.

Using Eqs. (26)–(28), we can generate the $r^* - P^*$, $\varepsilon - p^*$ and $\varepsilon - r^*$ curves, as shown in Figs. 2–4, respectively, where $r^* = r/kT_h$ and $p^* = p/kT_h$ are the dimensionless cooling rate and power input.

It is seen from Fig. 2 that the cooling rate is not, in general, a monotonic function of the power input, so there exists a maximum value for the cooling rate. It can be derived from Eq. (27) that when

$$y = \frac{d + e}{b + e} T_h / T_c, \quad (29)$$

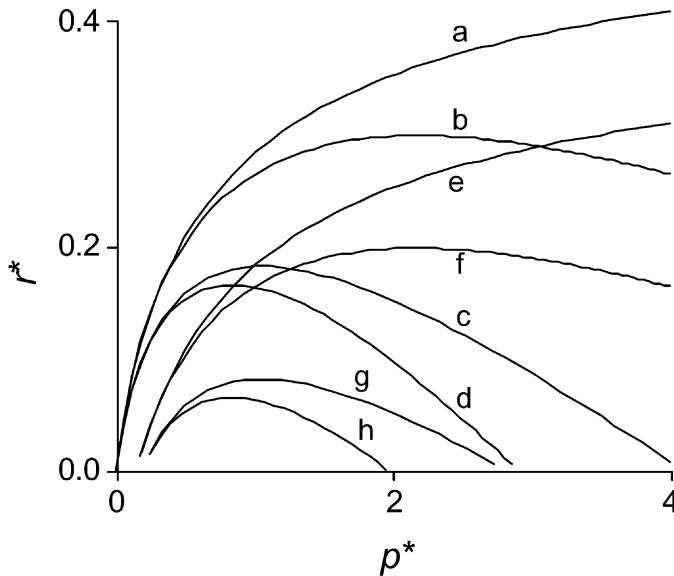


Fig. 2. The dimensionless cooling rate r^* versus power input p^* . Curves a ($q_L = 0$, $b = 0$, $e = 0$), b ($q_L = 0$, $b = 0$, $e = 0.05$), c ($q_L = 0$, $b = 0.1$, $e = 0$), d ($q_L = 0$, $b = 0.1$, $e = 0.05$), e ($q_L = 0.1$, $b = 0$, $e = 0$), f ($q_L = 0.1$, $b = 0$, $e = 0.05$), g ($q_L = 0.1$, $b = 0.1$, $e = 0$), and h ($q_L = 0.1$, $b = 0.1$, $e = 0.05$) are plotted for $T_h/T_c = 2$.

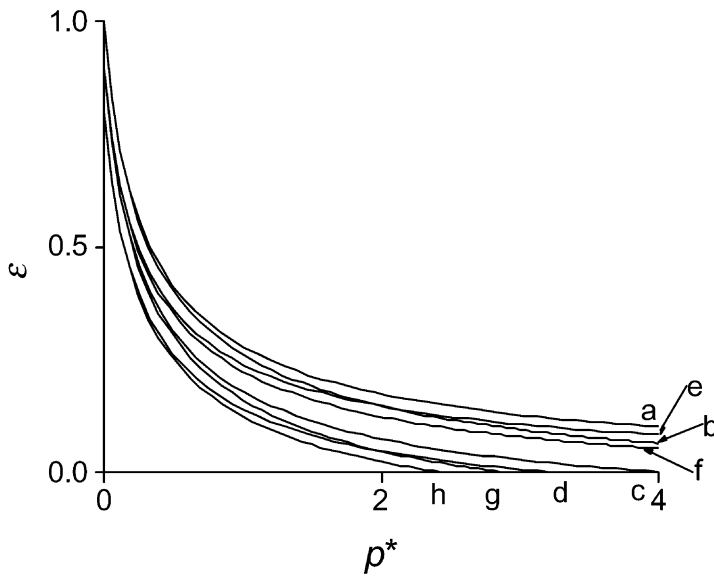


Fig. 3. The COP ε versus power input p^* curves. Curves a ($a/T_h = 0$, $b = 0$, $e = 0$), b ($a/T_h = 0$, $b = 0$, $e = 0.05$), c ($a/T_h = 0$, $b = 0.1$, $e = 0$), d ($a/T_h = 0$, $b = 0.1$, $e = 0.05$), e ($a/T_h = 0.05$, $b = 0$, $e = 0$), f ($a/T_h = 0.05$, $b = 0$, $e = 0.05$), g ($a/T_h = 0.05$, $b = 0.1$, $e = 0$), and h ($a/T_h = 0.05$, $b = 0.1$, $e = 0.05$) are plotted for $T_h/T_c = 2$.

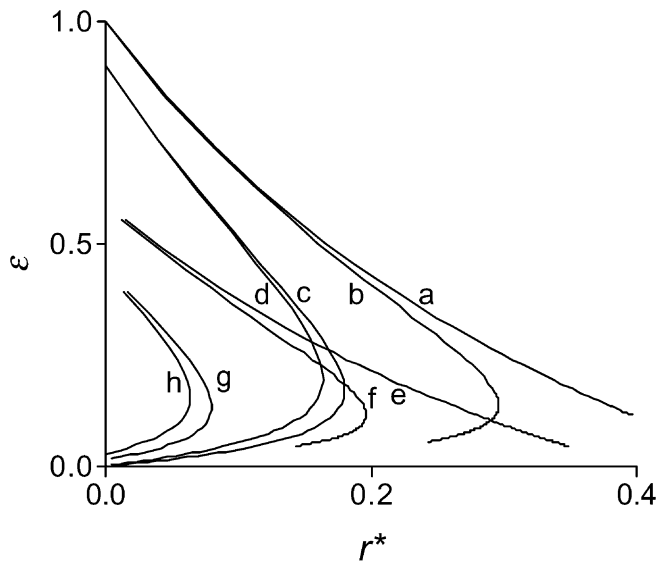


Fig. 4. The COP ε versus dimensionless cooling rate r^* curves. The values of the parameters T_h/T_c , a/T_h , b and e are the same as those used in Fig. 3. Curves a, b, c, d and e, f, g, h correspond to the cases of $q_L = 0$ and 0.1, respectively.

the cooling rate attains its maximum value, i.e.,

$$r_{\max} = kT_c \frac{\frac{(b+e)T_c}{(d+e)T_h - (b+e)T_c} - b}{\frac{(b+e)(d+e)T_c}{(d-b)[(d+e)T_h - (b+e)T_c]} + e} - q_L, \quad (30)$$

where $d = \sqrt{(1+b)(b+e)T_c/T_h - be}$. In such a case, the corresponding power-input and coefficient of performance are

$$p = \frac{kT_c}{\frac{(b+e)(d+e)T_c}{(d-b)[(d+e)T_h - (b+e)T_c]} + e} \equiv p_r \quad (31)$$

and

$$\varepsilon = \frac{(b+e)T_c - a(1+h)(b+e)(d+e)/[d+e+h(b+e)]}{(d+e)T_h - (b+e)T_c} - b \equiv \varepsilon_r, \quad (32)$$

and the corresponding temperatures of the working substance in the two isothermal processes are

$$T_1 = T_h \frac{d+e+h(b+e)}{(1+h)(b+e)} \equiv T_{1r} \quad (33)$$

and

$$T_2 = T_c \frac{d+e+h(b+e)}{(1+h)(d+e)} \equiv T_{2r} \quad (34)$$

It is seen from Fig. 3 that the coefficient of performance is a monotonically decreasing function of the power input. When the power input is equal to p_ε , the coefficient of performance reduces to 0. From Eqs. (26) and (28), one obtains an explicit expression for p_ε as

$$p_\varepsilon = kT_c \frac{y_\varepsilon - 1}{y_\varepsilon/(y_\varepsilon - T_h/T_c) + e(y_\varepsilon - 1)} \quad (35)$$

where

$$y_\varepsilon = \left\{ (1+b)T_c - a(1+h) - bhT_h \right. \\ \left. + \sqrt{[(1+b)T_c - a(1+h) - bhT_h]^2 + 8(1+b)bhT_hT_c} \right\} / (2bT_c).$$

Thus, the region of $p \geq p_\varepsilon$ is not allowed for a ferroelectric Stirling refrigeration cycle.

It is seen from Fig. 4, that when the regenerative losses and regenerative time are negligible, $b = 0$ and $b_1 = 0$. The coefficient of performance is a monotonically decreasing function of the cooling rate. This is similar to the case of a Carnot refrigeration cycle, in which the time of the adiabatic processes is negligible compared with that of the isothermal processes. For other cases, the coefficient of performance is not a monotonically decreasing function of the cooling rate. There exists a maximum cooling rate, which is given by Eq. (30). When $r < r_{\max}$, there are two different coefficients-of-performance for a given cooling rate, where one is smaller than ε_r and the other is larger than ε_r . Obviously, the region of $\varepsilon < \varepsilon_r$ is not optimal because the coefficient of performance decreases with the decrease of the cooling rate. For a ferroelectric Stirling refrigeration-cycle, the optimal region of the coefficient of performance should be determined by

$$\varepsilon \geq \varepsilon_r. \quad (36)$$

According to Eq. (36), we may further determine the suitable ranges of the power input and the temperatures of the working substance in the two isothermal processes as

$$p \leq p_r, \quad (37)$$

$$T_1 \leq T_{1r}, \quad (38)$$

and

$$T_2 \geq T_{2r}. \quad (39)$$

Obviously, the parameters $r_{\max}$, ε_r , p_r , T_{1r} and T_{2r} are very important and may provide some criteria for the optimal design and operation of a ferroelectric Stirling refrigeration cycle.

5. Discussion

5.1. Rapid regenerative processes

When $u = 0$, this implies that the time of the two regenerative processes is negligible compared with that of the two isothermal processes, although the regenerative losses are taken into account. In this case, the $r^* - P^*$, $\varepsilon - p^*$ and $\varepsilon - r^*$ curves are shown as the curves labelled g in Figs. 2–4 and the above equations may be simplified. For example, Eqs. (27)–(34) may be written as

$$r = kT_c \frac{1 - b(y - 1)}{y/(y - T_h/T_c)} - q_L, \quad (40)$$

$$p = kT_c \frac{y-1}{y/(y-T_h/T_c)}, \quad (41)$$

$$y = \sqrt{(1+b)T_h/(bT_c)}, \quad (42)$$

$$r_{\max} = kT_c(\sqrt{1+b} - \sqrt{bT_h/T_c})^2 - q_L, \quad (43)$$

$$p_r = k[\sqrt{(1+b)T_c/(bT_h)} - 1][T_h - \sqrt{bT_hT_c/(1+b)}], \quad (44)$$

$$\varepsilon_r = \frac{T_c}{\sqrt{(1+1/b)T_hT_c} - T_c} - \frac{a(1+h)}{\sqrt{(1+1/b)T_hT_c} - T_c} \frac{\sqrt{(1+1/b)T_c/T_h}}{\sqrt{(1+1/b)T_c/T_h} + h} - b, \quad (45)$$

$$T_{1r} = T_h \frac{\sqrt{(1+b)T_c/(bT_h)} + h}{1+h}, \quad (46)$$

and

$$T_{2r} = T_c \frac{1+h\sqrt{bT_h/[(1+b)T_c]}}{1+h}. \quad (47)$$

5.2. Highly efficient regenerative processes

When $\eta \rightarrow 1$, the regenerative losses in the two constant electric polarization processes are negligible. In such a case, the $r^* - P^*$, $\varepsilon - p^*$ and $\varepsilon - r^*$ curves are shown as the curves labelled f in Figs. 2–4 and Eqs. (26), (27) and (29)–(34) may be simplified as

$$\varepsilon = \frac{T_c y + h(T_h - ay) - ay}{(y-1)(T_c y + hT_h)}, \quad (48)$$

$$r = kT_c \frac{1}{y/(y-T_h/T_c) + e(y-1)} - q_L, \quad (49)$$

$$y = (1+d/e)T_h/T_c, \quad (50)$$

$$r_{\max} = \frac{kT_c}{1-e+eT_h/T_c+2\sqrt{eT_h/T_c}} - q_L, \quad (51)$$

$$p_r = kT_c \frac{[1+\sqrt{T_c/(eT_h)}]T_h/T_c - 1}{1-e+eT_h/T_c+2\sqrt{eT_h/T_c}}, \quad (52)$$

$$\varepsilon_r = \frac{T_c - a(1+h)[1 + \sqrt{T_c/(eT_h)}]/[1 + h + \sqrt{T_c/(eT_h)}]}{[\sqrt{T_c/(eT_h)} + 1]T_h - T_c}, \quad (53)$$

$$T_{1r} = T_h \frac{1 + h + \sqrt{T_c/(eT_h)}}{1 + h}, \quad (54)$$

and

$$T_{2r} = T_c \frac{1 + h + \sqrt{T_c/(eT_h)}}{(1+h)[1 + \sqrt{T_c/(eT_h)}]}. \quad (55)$$

It should be pointed out that such a ferroelectric Stirling refrigeration-cycle with perfect regeneration has the same performance characteristics as an endoreversible ferroelectric Carnot refrigeration-cycle in which the time of the adiabatic processes is also determined by Eq. (18). Thus, the optimal performance of an endoreversible ferroelectric Carnot refrigeration cycle can be directly derived from the above results.

5.3. Rapid and highly efficient regenerative processes

When $u = 0$ and $\eta = 1$, the $r^* - P^*$, $\varepsilon - p^*$ and $\varepsilon - r^*$ curves are shown as the curves labelled e in Figs. 2–4 and Eq. (40) or (49) may be further simplified as

$$r = kT_c \frac{1}{y/(y - T_h/T_c)} - qL. \quad (56)$$

It is seen from curve e in Figs. 3 and 4 that the coefficient of performance is a monotonically decreasing function of not only the power input but also the cooling rate. When $y \rightarrow \infty$, $P \rightarrow \infty$, $\varepsilon = 0$, and $r = kT_c - qL$. It is clear that the cooling rate of a refrigerator is smaller than kT_c .

5.4. Zero heat-leak

When $Q_L = 0$, the $r^* - P^*$, $\varepsilon - p^*$ and $\varepsilon - r^*$ curves are shown as the curves labelled d in Figs. 2–4 and Eqs. (26), (27), (29), (30) and (32) may be simplified as

$$\varepsilon = \frac{1}{y - 1} - b, \quad (57)$$

$$r = kT_c \frac{1 - b(y - 1)}{y/(y - T_h/T_c) + e(y - 1)}, \quad (58)$$

$$r_{\max} = kT_c \frac{\frac{(b+e)T_c}{(d+e)T_h - (b+e)T_c} - b}{\frac{(b+e)(d+e)T_c}{(d-b)[(d+e)T_h - (b+e)T_c]} + e}, \quad (59)$$

and

$$\varepsilon_r = \frac{(b+e)T_c}{(d+e)T_h - (b+e)T_c} - b. \quad (60)$$

5.5. Other expressions of the polarization

When $\beta=0$, Eq. (1) may be simplified as

$$P = N\mu \tanh \frac{\mu E}{k_B T}. \quad (61)$$

At high temperatures, $\frac{\mu(E + \beta P)}{k_B T} \ll 1$ and $\frac{\mu E}{k_B T} \ll 1$. Eqs. (1) and (61) return to the Curie–Weiss law [24–27]

$$P = \frac{CE}{T - T_0} \quad (62)$$

and the Curie Law [5,24,25,27]

$$P = CE/T. \quad (63)$$

respectively, where $T_0 = N\mu^2\beta/k_B$ is the Curie temperature and $C = N\mu^2/k_B$ is a constant characteristic of the ferroelectric materials called the Curie constant. This implies the fact that when the working substance obeys the Curie–Weiss law and the Curie Law, the optimal performance of a ferroelectric Stirling refrigeration cycle can be directly derived from the results in this paper.

6. Conclusions

We have established an irreversible cycle model of the Stirling refrigerator using a ferroelectric material as the working substance, based on a general expression of the polarization of ferroelectric materials and a linear heat-transfer law. The influences of irreversibilities due to thermal resistances between the working substance and the heat reservoirs, regenerative losses in the two regenerative processes, and heat leak

loss between the heat reservoirs on the performance of the ferroelectric Stirling refrigeration cycle have been investigated. The maximum cooling rate and other relevant performance parameters are calculated. Some parameter criteria, which are helpful for the optimal design and operation of ferroelectric refrigerators, are given. The results obtained are quite general. The optimal performance of not only the ferroelectric Stirling refrigeration cycle but also the ferroelectric Carnot refrigeration cycle affected by the finite-rate heat transfer may be directly derived from the results in this paper as long as the dielectric materials, obeying Eq. (1), (61), (62), or (63), are used as the working substance.

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